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Optimal Task Assignment

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Abstract

This paper studies optimal task assignments in a risk neutral principal-agent model in which agents are compensated according to an aggregated performance measure. The main trade-off involved is one in which specialization allows the implementation of any possible effort profile, while multitasking constraint the set of implementable effort profiles. Yet, the implementation of any effort profile in this set is less expensive than that under specialization. The principal prefers multitasking to specialization except when tasks are complements and the output after success is small enough so that it is not second-best optimal to implement high effort in each task. This result is robust to several extensions such as the existence of multiple performance measures.

Keywords: Moral Hazard, Specialization, Multitasking, Implementation
JEL: J41, J24, D21.

1. Introduction

The question of how to allocate tasks among different workers to increase productivity was first answer in *The Wealth of Nations* by Adam Smith and its well exemplified by his description of how pins should be manufactured. This idea paved the way for the "scientific management" philosophy set forth a century ago by Frederick W. Taylor (1911). The basic idea was to view the task assignment problem as a scientific optimization problem, where industrial engineers study the production process and devise the most efficient way to break that process into individual, precisely defined tasks. Economists and psychologists have enriched the theory of task assignments in many different ways such as the role of comparative advantages, incentives, communication failures, motivation and learning problems and coordination costs among others.

In this paper, I revisit the question of what is the optimal task assignment in a setting where there are moral hazard and limited liability. More specifically, the paper considers a combinatorial agency model in which there is one project whose expected output depends on the effort exerted in several tasks. The outcome of the project can be either success or failure and the probability of success allows for both complementary as well as substitutable tasks. The principal chooses between two different task assignments: multitasking, which means that all tasks are assigned to one agent; and specialization or team work, which means that each task is assigned to a different agent. The principal is risk-neutral, agents are effort averse and face a limited-liability constraint. Effort is dichotomic (high and low effort) and unobservable in each task, and agents' marginal cost of effort is constant and identical in each task.

In the absence of moral hazard, the principal is completely indifferent between specialization and multitasking. The reason stands for the fact that, regardless of the technology and task assignment chosen, the first-best efficient effort profile, which is assumed to be high effort in each task, can be implemented at no extra cost. Hence, the environment

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proposed here has been deliberately kept as simple as possible so that if the principal shows strict preferences for a particular task assignment that must be due to incentive considerations only.

When there is moral hazard and tasks are substitutes, the principal prefers multitasking to specialization. The effort profile implemented depends on magnitude of the return upon success; the higher is this, the higher the effort profile implemented. In contrast, when tasks are complements, multitasking dominates specialization only when it is second-best optimal to induce the agent in charge of all tasks to work hard in each of them. That is, when the return after success is sufficiently large. Otherwise, the principal adopts a specialized task assignment and the effort profile implemented increases as the magnitude of the return after success rises. Hence, specialization or team work arises as the optimal task assignment only if tasks are complements.

The optimal assignment is determined by the following forces. First, task complementarity together with the fact that compensation is based on an aggregated performance measure that confounds the efforts of n non-conflicting tasks implies that there is no contract that implements an effort profile where the agent responsible for all tasks works in less than n tasks and in at least one of them. When the incentive intensity provided to the agent is such that he works hard in one task, he has incentives to work hard in the other tasks as well since the marginal gain from effort rises with the effort exerted in other tasks, while the marginal cost is constant. In other words, the global incentive constraint is the relevant one to determine the effort profile that can be implemented. Second, when tasks are substitutes, the incentive intensity that induces an agent to work hard in k tasks, it does not induce him to work hard in $k+1$ tasks since the marginal gain from effort falls with the effort exerted in other tasks, while the marginal cost is constant. In other words, the downward local incentive constraints are the relevant ones to determine the agent's optimal effort profile. Third, when a different agent is responsible for each task (i.e., specialization is adopted), the principal can implement any effort profile he wishes as a Nash equilibrium. This can be done since the principal has the freedom to customize each agent's incentive intensity so that incentives for hard work given to one agent do not induce other agents to work hard. Fourth, the limited liability rent needed to implement any effort profile is always at least as large in the case of specialization as it is in the case of multitasking. The reason is twofold: first, an agent's compensation is based on an aggregated performance measure that confounds the effects of several non-conflicting tasks. This induces the agent responsible for several tasks to internalize the losses from not exerting effort in any given task for whom he is responsible for; and second, under specialization the bonus must be paid k times in order to induce k agents to work hard.

When tasks are complements, the interaction of these factors results in a trade-off between paying a lower limited liability rent by making an agent responsible for every task, but restricting the set of implementable effort profiles and paying a higher aggregated limited liability rent by allocating each task to a different agent and being able to implement any effort profile. This trade-off has been overlooked in the literature since all the papers I am aware-off assume that implementing high effort in each task is optimal regardless of the cost that this entails and the number of tasks involved. In contrast, when tasks are substitutes, this trade-off does not arise since multitasking does not restrict the set of implementable effort profiles. Regardless of the task assignment chosen, the principal can implement any effort profile he wishes at a lower cost under multitasking. The reason stands for the fact that when tasks are substitutes and a multi-task assignment is chosen, the implementation of any effort profile is determined by the local incentive constraints, which are identical to those that arise when a specialized task assignment is chosen. This gives rise to the same limited liability rent, but under specialization this is paid k times (k being the number of tasks for which the principal wishes to implement high effort) while under multitasking this is paid just once.

This result is shown to be robust to the fact that effort in a given number of tasks is contractible, to the fact that agents might be limited in their ability to work hard in more than a given number of tasks and to the presence of multiple performance measures. I show that the first two constraints increase the set of parameters under which multitasking is optimal since the former results in a lower limited liability rent under multitasking, while the latter increases the set of implementable effort profiles under multitasking. Multiple performance measures do not solve the implementation problem that arises under multitasking, but consistent with the informativeness principle, result in lower total compensation costs.

Related Literature.. The optimal assignment of tasks in principal-agent problems with moral hazard has been studied by several authors such as Holmström and Milgrom (1991), Baker (2002), Itoh (1991, 1992, 1994), Dewatripont et al. (1999), Zhang (2003), Corts (2007), Mukherjee and Vasconcelos (2011) and MacDonald and Marx (2001) in a setting where the effort substitution approach pioneered by Holmström and Milgrom (1991) plays a crucial role.[2] However, in the current paper, the multitasking problem is not based on the effort substitution approach and how the number of tasks affects the risk and incentives trade-off under substitution, it is rather based on the ideas that tasks complementarities create implementation problems when several tasks are bundled into the same job. Thus, in what follows I will discuss only this less known strand of the multitasking literature.

In terms of the type of multitasking problem studied here, this article is more closely related to the contributions of Laux (2001), Chen (2012), Zhao (2008), Ratto and Schnedler (2008) and Dewatripont and Tirole (1999). Laux (2001) provides a rationale for multitasking. He analyzes a multi-task agent model in which the agent's effort choice on each task is binary, effort costs are linear, tasks are independent from each other and there is one performance measure per task.[3] He provides a rationale for multitasking as an optimal job design. Mainly, he shows that incentive problems are a natural source of economies of scope in the sense that allocating multiple tasks to a single agent relaxes the agent's limited-liability constraint. The main consequence of this is that it might be optimal to increase the scope of the job with the natural consequence that the agent may exert an inefficiently high amount of total effort. Thus, multi-tasking arises as a mechanism to lower the agent's limited-liability rent and the implementation problem is not considered since he focus on the equilibrium in which high effort is chosen in each task.[4]

Ratto and Schnedler (2008) provide an explanation for the optimality of the division of labor similar to the one here. They study a situation where production requires two non-conflicting tasks, and the manager wants to direct production to achieve a preferred allocation of effort across tasks. However, aggregated production is the only indicator of agent activity. The main result is that the principal cannot implement the preferred allocation with a single agent, yet he is able to do so by inducing a game among two agents. They show that when a principal bases agents' compensation on an aggregated and contractible output, cares about how a given output is achieved and tasks are asymmetric, multitasking precludes the implementation of the desired effort level that leads to the required output. Mainly, the agent engages in *window dressing*, which means that he switches effort from the hardest task to the easiest in order to produce the principal's desired output at the lowest possible private cost for him. In contrast, specialization allows the principal to implement the desired effort level as a unique Nash equilibrium. They argue then that specialization could be optimal from the principal's viewpoint. Their rationale is similar to the one here, multitasking precludes the implementation of certain effort profiles and this could make specialization optimal under certain circumstances. The multitasking problem though is of a different nature. Mainly, here it occurs because of task complementarities and not because the performance measure provides room for window-dressing. Furthermore, their result requires asymmetric tasks, while mine holds with both symmetric and asymmetric tasks.

Dewatripont and Tirole (1999) also provide a rationale for specialization but based on direct conflicts between tasks. They show that it always better to split the task of finding evidence in favor and against a decision between two

[2] Zhang (2003) shows, in contrast to the result here, that in the absence of task complementarities, specialization always dominates multitasking. The difference in the result stands for the fact that in his model tasks differ according its measure difficulty and therefore bundling different tasks within the same job worsen the noisiness of the aggregate signal with respect to the case in which in each job identical tasks are bundled. The noisiness of the signal matters in his model because agents are risk averse, while here agents are risk neutral and this plays no role.

[3] The optimal contract depends on the number of success and the optimal contract pays a positive bonus only when the maximum number of successes is realized. From the agent's viewpoint, this makes effort in one task complementary to effort in other tasks.

[4] Bond and Gomes (2009) generalize the model in Laux (2001) by considering contracts in which an upper bound on payments and monotonicity constraints are imposed, the agent's effort choice on each task is continuous, and the production function is non-linear. They identify a different source of allocation inefficiency across tasks, which is that under the optimal contract, it could be optimal for the agent to focus only on a sub-set of tasks, and its consequence is that there is insufficient total effort.

agents. The reason stands for the fact that the optimal compensation can be based only on an aggregated measure of the task and this is increasing in the outcome of one task and decreasing in the outcome of the other task. This implies that it is impossible to induce one agent to exert more effort in both tasks and thus it is optimal to split the task between two different agents to avoid conflicts of interest in job design. The main difference in their set-up and mine is that I do not consider conflicting tasks. With conflicting tasks, incentives create competition between agents, while with non-conflicting tasks, agents are aligned in terms of their contribution to the performance measure.[5]

Finally, the structure of the model used here is very similar to that in Winter (2004, 2009, 2010). However, the issues studied in those paper are completely different than the issue study in this paper despite the fact that some of the results for the case of specialization in this paper resemble those on Winter (2004). Mainly, he focuses on how unique implementation of the effort profile in which each agent exerts effort results in discrimination in the sense that each identical agent is paid a different bonus. While in the case of specialization here we have multiple Nash equilibria, the unique implementation result is of no help since we are concerned not only with the cases in which effort is exerted in each task, but also on those where it is optimal for the principal that some agents exert effort and others do not.

2. The Model

Lets consider a model in which a principal hires a variable number of agents (workers), indexed by $j \in \{1, \ldots, J\}$, to accomplish a finite number of tasks, indexed by $i \in \{1, \ldots, n\}$, with $n \geq 2$. The firm entails an endogenous number of jobs, each job requires one agent and jobs can be set up to be accomplished with a variable number of tasks. There are J^n task assignments that the principal may choose, but here I will focus on only two of them; (i) full multitasking: the principal hires one agent and makes him responsible for tasks 1 to n; and (ii) full specialization or team work: the principal hires n agents and delegates task j to agent i. While at first glance this may seem overly restrictive, considering these two assignments provides sufficient complexity for one to go on to more general task assignments and it is enough to emphasize the main incentive trade-offs studied in this article.[6]

Tasks and Technology.. The agent responsible for task i must choose an effort level belonging to the set of efforts E_i and doing so entails a cost of effort ce_i for each possible effort $e_i \in E_i$.[7] To keep the analysis tractable, the action space for each task is binary: in each task the agent responsible for it chooses between action 0 (low effort) and action 1 (high effort). Action 0 should be interpreted as the level of effort that can be implemented without offering explicit incentives rather than no effort or in as the status decision while action 1 as an innovation.

The outcome space has only two states success and failure. Each state has associated a return or output: the output in the success state is given by v and that in the failure state is 0. The effort profile $e \equiv (e_1, \ldots, e_n)$ determines the probability distribution over the outcome space according to the success function:

$$p(e) : \{0, 1\}^n \to [0, 1].^{8}$$

In what follows I will assume that tasks are symmetric and define $\Delta(e_{-i}) \equiv p(1, e_{-i}) - p(0, e_{-i})$. Because the focus here is on the incentives for effort rather than coordination issues among agents, the probability of success satisfies $\Delta(e_{-i}) > 0, \quad \forall e_{-i} \in \{0, 1\}^{n-1}$. Additionally, I assume that $p(e) > 0, \forall e \in \{0, 1\}^n$. That is the effort profile in which low effort is chosen in each task yields a positive expected output.

[5] Conflicting tasks also provide incentives for sabotage and collusion that are unavoidable, which is not necessarily the case with non-conflicting tasks.

[6] In fact Emek and Feldman (2009) show that when tasks are complements the number of possible job design grows exponentially with the number of tasks. In other words, the problem of computing the optimal contract in which tasks are complements is NP-hard. Hence, the task of characterizing the optimal task assignment when one does not restrict the set of choices is likely to be unsurmountable.

[7] In what follows I will use the terms effort and action indistinguishable.

[8] The simplest version of this general formulation is the case in which $p(e) = p(\sum_{i=1}^{n} e_i)$. However, adopting this simple formulation neither makes the proofs simpler nor it allows us to gain further intuition.

Given two effort profiles $e', e \in \{0,1\}^n$, I denote $e' < e$ if for every agent i it holds that $e'_i \leq e_i$ and for some i, $e'_i < e_i$.

An important characteristic of the success function in the forthcoming analysis is the relationship between tasks.

Definition 1.

i) *Tasks are strict complements (SSPM) if for every i and every $e'_{-i} < e_{-i}$, it holds that $\Delta(e_{-i}) > \Delta(e'_{-i})$.*

ii) *Tasks are strict substitutes (SSBM) if for every i and every $e'_{-i} < e_{-i}$, it holds that $\Delta(e_{-i}) < \Delta(e'_{-i})$.*[9]

iii) *Tasks are independent (IND) if for every i and every $e'_{-i} < e_{-i}$, it holds that $\Delta(e_{-i}) = \Delta(e'_{-i})$.*

Intuitively SSPM means that the marginal contribution of effort in any task to the probability of success increases with the effort exerted in another tasks; i.e., efforts are complements, while SSBM means that the marginal contribution of effort in any task to the probability of success decreases with the effort exerted in another tasks; i.e., efforts are substitutes. An IND technology is one in which the marginal return to effort in each task is independent of the effort chosen in any other task; i.e., efforts are neither complements nor substitutes.

These definitions can be applied to any function of the effort profile e. So in the forthcoming analysis when I say, for instance, that the agent j's expected utility satisfies SSPM, it means that from the agent's point of view efforts are complements or his expected utility function is strictly supermodular in e. Similarly, for properties SSBM and IND.

Contracts and Payoffs.. Because actions are not observed, but output is contractible, the principal can design enforceable output-based contracts. Because the outcome space is binary, a contract is a tuple (α, β), where α is paid regardless of the observed outcome and β is a non-negative bonus paid when success is observed. Thus, total compensation is $\alpha + \beta$ when success is observed and α when failure is observed. This payment scheme evidently induces externalities among agents; ceteris-paribus, each agent will benefit if a colleague works harder (chooses high effort) and lose income if a colleague shirks (chooses low effort). However, the externality could be such that the the marginal impact of the effort allocated to any given task is independent of the effort allocated to a different task.[10]

Lets define E_j as agent j's effort choice set and notice that $E_j = \{0,1\}^n$ under multitasking and $E_j = E_i$ under full specialization in task i. Then, when agent j faces contract $w_j \equiv (\alpha_j, \beta_j)$, the chosen profile is e and agent j chooses the effort profile e_j, his utility is given by:

$$U_j(w_j, e) \equiv \alpha_j + p(e)\beta_j - c \sum_{e_i \in E_j} e_i. \tag{1}$$

Each agent's reservation payoff is exogenously given and equal to $U_j = 0$ and he is wealth constraint and therefore payments from the agents to the principal are not allowed.

Let $w = ((\alpha_1, \beta_1), \ldots, (\alpha_n, \beta_n))$. The principal's expected payoff is as follows:

$$V(w, e) \equiv p(e)v - \sum_{j=1}^{J} \left(\alpha_j + p(e)\beta_j\right).$$

Lets denote the effort profile where high effort is exerted in k tasks and low effort is exerted in the remaining $n-k$ tasks by 1_k.[11] Then, I will assume the following parametric restrictions:

Assumption 1.

$$p(1_n)v - cn \geq p(1_k)v - ck, \ \forall k \in \{0, 1, \ldots, n\}$$

[9] SSPM stands for strict supermodularity and SSBM stands for strict submodularity.

[10] See, for instance, Baker (2000), Prendergast (2008) and Corts (2007) for papers studying multitasking under this last restriction.

[11] The possible permutations are irrelevant since tasks are symmetric. One can deal with asymmetric tasks at a higher algebraic burden without gaining in intuition.

Hence, the highest total welfare is achieved when high effort is exerted in each task and, the allocation of tasks to agents is irrelevant when the efficient effort profile can be implemented at no extra cost.[12]

The environment proposed here has been deliberately kept as simple as possible so that in the absence of moral hazard, the principal would be completely indifferent with regard to any task assignment. Hence, if I find strict preferences of the principal for a particular assignment, that must be due to incentive considerations only.

3. Preliminaries

3.1. Effort Choices

3.1.1. Specialization

Given a task assignment in which each task is the responsibility of a different agent, agent j, who is responsible for task i, chooses effort $e_i = 1$ if and only if

$$U_j(w_j, 1, e_{-i}) \geq U_j(w_j, 0, e_{-i}).$$

It readily follows from this that for any effort profile e_{-i}, agent j chooses to work hard in task i if and only if

$$\beta_j \geq \frac{c}{p(1, e_{-i}) - p(0, e_{-i})}. \tag{2}$$

The marginal benefit to agent j of working hard in task i consists of an increase in the expected performance-contingent compensation $\beta_j \triangle(e_{-i})$ and the marginal cost of doing so is the increase in the the cost of effort c.

Lets define B_k as $\frac{c}{p(1_k)-p(1_{k-1})}$. Then when $p(e)$ is such that efforts are complements and contracts are such that $\beta_j \geq B_k$ for $j \in \{1, \ldots, k\}$ and $\beta_j < B_{k+1}$ for $j \in \{k+1, \ldots, n\}$ for any $k \in \{1, \ldots, n\}$, there are two non-outcome equivalent pure strategy Nash equilibria: one where k agents work hard and one in which none of them works hard. The intuition is that the effort choice game becomes a coordination game. In contrast, when $p(e)$ is such that tasks are substitutes, there is a unique equilibrium in pure strategies.

When $p(e)$ satisfies SSPM and therefore $U_j(w_j, e)$ is supermodular, the game exhibits positive spillovers (that is, the payoff to a player is increasing in the strategies of the other players) and therefore the largest (smallest) equilibrium point is the Pareto best (worst) equilibrium (see, Milgrom and Roberts (1990)). Thus, if I restrict myself to the Pareto best equilibrium point,[13] it is easy to show the following:

Lemma 1. *Suppose agent j is responsible for task i only. Then, if $\beta_j \geq B_k$ for $j \in \{1, \ldots, k\}$ and $\beta_j < B_{k+1}$ for $j \in \{k+1, \ldots, n\}$, there exists a Nash equilibrium in the effort subgame given by: $e^* = 1_k$.*

3.1.2. Multitasking

Consider here the case in which agent j is responsible for each one of the n tasks. Then, agent j will choose the effort profile e if and only if

$$U_j(w_j, e) \geq U_j(w_j, e'),\ \forall e' \in \{0, 1\}^n.$$

[12] If agents were heterogeneous and tasks sensitive to this heterogeneity, then the task assignment would no longer be irrelevant in the sense that who is allocated to which task would matter.

[13] I could deal with this multiple equilibria issue by focusing on a team equilibria as defined by Che and Yoo (2001); that is, the equilibrium that is most favorable to agents in the sense that the chosen equilibria yields the highest joint payoff and thus it eliminates agents' incentives to collude. In this setting this requires to set $\beta_j = B(1_0)$ for each $j \in J$. Following Winter (2004), I could also focus on unique implementation by allowing the principal to discriminate agents in terms of the contract offered to each agent. Doing so one can show that the total cost of implementing any effort profile is higher under unique implementation since the contract offered to each agent must be such that low effort is strictly dominated by high effort for any possible effort profile.

When $p(e)$ satisfies SSPM, the marginal impact of the agent's effort in one task on his expected utility rises with the effort he exerts in other tasks. This implies that the agent prefers high effort in $k \geq 1$ tasks and low effort in the rest to high effort in $k-1$ tasks and low effort in the remaining tasks, then he prefers high effort in each task to high effort in k tasks and low effort in the rest. Furthermore, this also implies that if the agent prefers high effort in each task to low effort in each of them, he also prefers high effort in each of them to any other effort profile.[14] Hence, the agent prefers high effort in each task to low effort in each of them if and only if[15]

$$\beta_j \geq B_{sspm} \equiv \frac{cn}{p(1_n) - p(1_0)}. \tag{3}$$

Otherwise he prefers low effort in each task.

When $p(e)$ satisfies SSBM, the marginal impact of the agent's effort in one task on his expected utility falls with the effort he exerts in other tasks. This implies that if effort incentives are such that he prefers high effort in one task and low effort in the rest to low effort in each task, he may or may not prefer high effort in each task. In fact, one can easily check that the agent's choice of effort is fully determined by the adjacent or local incentive constraints. The reason stands for the fact that the marginal return to effort falls as the agent exerts more effort on other tasks. Hence, it is easy to show that he prefers to work hard in $k \in \{1, \ldots, n\}$ tasks over any other possible effort profile if and only if[16]

$$\beta_j \geq B_k \equiv \frac{c}{p(1_k) - p(1_{k-1})} \tag{4}$$

and

$$\beta_j < B_{k+1} \equiv \frac{c}{p(1_{k+1}) - p(1_k)}. \tag{5}$$

When tasks are independent (i.e., $U_j(w_j, e)$ satisfies IND), the principal can implement any effort profile he wishes under the assumption that the agent chooses the principal's preferred effort profile. Yet, this is highly unstable equilibrium since a little perturbation in either the cost of effort or the bonus results in that the agent either chooses high effort in each task or no effort in each of them. Hence, for the sake of brevity I will consider the case of independent tasks together with the case of complementary tasks.

The following result follows from the discussion above.

Lemma 2. *Suppose agent j is responsible for every task $i \in \{1, \ldots, n\}$. Then,*

i) *if $p(e)$ satisfies SSPM or IND, the optimal effort profile is given by: $e_m = 1_n$ if $\beta_j \geq B_{sspm}$ and $e_m = 1_0$ if $\beta_j < B_{sspm}$; and*

ii) *if $p(e)$ satisfies SSBM, the optimal effort profile is given by: $e_m = 1_k$ if $B_{k+1} > \beta_j \geq B_k$ for any $k \in \{1, \ldots, n\}$ and $e_m = 1_0$ if $\beta_j < B_1$.*

The intuition for this result can can better seen in figure 1.

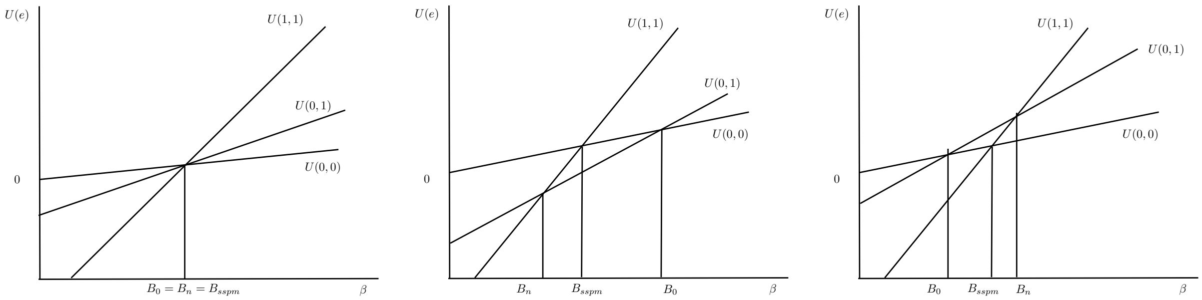
When tasks are independent (panel (a) figure 1), from an agent's viewpoint the marginal return to effort in one task is independent of the effort level he exerts in other tasks. This implies that his decision to work hard in one task is independent of the effort exerted in the other tasks. When the agent is offered a bonus strictly lower than B_{sspm}, he chooses low effort in each task, while when the bonus is strictly higher than B_{sspm}, he chooses high effort in each task. When the bonus is exactly equal to B_{sspm}, he is indifferent between any effort profile $1_k, \ \forall k \in \{0, \ldots, n\}$.

[14] This follows from noticing that SSPM implies that $p(1_n) > p(1_0) + n\big(p(1_1) - p(1_0)\big)$.

[15] Technically, the global incentive constraint implies the local incentive constraints.

[16] Technically, the local incentive constraints are not implied by the global incentive constraint.

Fig. 1. Incentive Compatibility



(a) $\Delta(1_k) = \Delta(1_{k-1})$ for all k (b) $\Delta(1_k) > \Delta(1_{k-1})$ for all k (c) $\Delta(1_k) < \Delta(1_{k-1})$ for all k

The reason for this result is twofold: first, the probability of success is aggregated and confounds the efforts of n non-conflicting tasks; and second, the probability of success and the agent's costs are such that for any incentive intensity, the agent perceives tasks as independent. Thus, when $\beta_j = B_{sspm}$ the marginal benefit of working hard in one task is exactly equal to the marginal cost of the extra effort in that task, regardless of the effort chosen in the other task. An economic consequence of the fact that under a multitasking job design the cost minimizing contract that implements any effort profile $1_k, \ \forall k \in \{0, \dots, n\}$ leaves the agent indifferent between this effort profile and any other effort profile $1_k, \ \forall k \in \{0, \dots, n\}$ is that small changes in economic fundamentals can have a big impact on the agent's effort level.[17] For instance, suppose that the cost of effort c unexpectedly rises by a small amount (i.e., from c to $c + \epsilon$, with ϵ as small as desired). After the change, the agent prefers 1_0 to all alternative effort profiles, and so switches to exerting no effort. In this sense, the agent's effort level is "fragile" and for this reason I treat the case of independent tasks together with the case of complementary tasks.[18] When tasks are complements (panel (b) figure 1); i.e., more effort in one task would be beneficial on the margin for his incentives on the other tasks, the agent will never choose high effort in some tasks and low effort in others. He will choose either high effort in each task or low effort in each of them. Hence, this complementarity results in an implementation problem in the sense that there is no contract able to implement an effort profile different from $\{1_0, 1_n\}$. In contrast, when tasks are substitutes (see, panel (c) figure 1), any effort profile could be implemented. This issue, while simple and intuitive, it has been overlooked in the literature since all papers I am aware-off focus on the case in which the returns are always high enough so that the implementation of the effort profile 1_n is optimal even when agents must be paid a limited liability rent in order to induce them to choose high effort.

[17] A similar result is found in Bond and Gomes (2009) but using a different model.

[18] It is natural to ask how the fragility implication would be affected if the principal were to take into account the potential change in parameters at the time contracts are offered. For instance, if the principal were aware that with some small probability, say ϕ, the cost of effort will increase by an amount ϵ and he modifies the contract at all, he will clearly do so by enough to ensure that effort does not collapse when the shock hits. However, doing so has a discrete cost. As such, if the probability ϕ is small enough, the principal will use the same contract as when the shock is completely unanticipated.

3.2. Optimal Contracts

Given a task assignment and effort profile e, the principal chooses compensation contracts to solve the following program

$$\min_{w\in\Re^{2J}_{+}} \Big\{ \sum_{j=1}^{J} \big(\alpha_j + p(e)\beta_j\big) \Big\} \tag{CMI}$$

subject to

$$e_j \in \operatorname*{argmax}_{e'_j\in E_j} U_j(w_j, e),\ \forall j \in J, \tag{ICj}$$

$$U_j(w_j, e) \geq 0,\ \forall j \in J, \tag{PCj}$$

$$\alpha_j \geq 0, \alpha_j + \beta_j \geq 0,\ \forall j \in J, \tag{LL}$$

where the constraint in equation (ICj) is agent j's incentive compatibility constraint and that in equation (PCj) is his participation constraint.

Because incentives are created only by the size of the bonus, the principal's objective function falls continuously with α_j and agent j's expected utility rises continuously with α_j, it is easy to show that it is optimal to set $\alpha_j = 0$ so that the limited liability constraint holds when the project fails. In addition, this together with the fact that agents choose effort optimally, imply that each agent's participation constraint is satisfied. Hence, program (CMI) can be written as follows

$$\min_{(\beta)\in\Re^{J}_{+}\times\{0,1\}^n} \Big\{ p(e) \sum_{j=1}^{J} \beta_j \Big\} \tag{CMII}$$

subject to

$$e'_j \in \operatorname*{argmax}_{e'_j\in E_j} U_j(w_j, e),\ \forall j \in J \tag{ICj}$$

$$\beta_j \geq 0,\ \forall j \in J. \tag{LL}$$

Let the solution to this problem be W_e. Then, the principal chooses the effort profile e that maximizes his expected utility; that is

$$\max_{e\in\times\{0,1\}^n} \{p(e)v - W_e\} \tag{PI}$$

Lets denote the optimal solution to this program when the principal chooses a specialized task assignment and it is optimal to implement the effort profile e by $V^s(e)$, and lets denote this when the principal chooses a multitask assignment and it is optimal to implement the effort profile e by $V^m(e)$. In the following sub-section, I fully characterize the solution to this problem; first for the full specialization case and then for the multitasking case.

3.2.1. Specialization.

The formal proofs of the results in this subsection are identical to those in Babaioff et al. (2012) and therefore for the sake of brevity they are omitted.

Lets define the lowest total compensation costs when $k \in \{0, \ldots, n\}$ agents are incentivized to work hard as the sum of the limited liability rents for those agents who are incentivized to choose high effort (the remaining agents are paid zero); that is,

$$W_k^s \equiv p(1_k)\sum_{j=1}^{k} B_k = \frac{p(1_k)}{\Delta(1_{k-1})}ck.$$

Notice that the aggregated limited liability rent increases with the number of agents who are motivated to choose high effort when tasks are substitutes or independent, while this may either rise or fall otherwise.

Lemma 3. *For any technology $p(e)$ each of the following is monotonically non-decreasing with the return v: (i) the expected utility of the principal evaluated at the optimal contracts; (ii) the success probability evaluated at the optimal contracts; and (iii) the expected payment evaluated at the optimal contracts*

This result shows that under the optimal contract, as the return for success rises, the principal shares this return with the agents in such a way that everyone is better-off. The reason stands for the fact that an increase in v increases the marginal return to effort and thus it is optimal to provide more agents with incentives for effort. The more technical intuition for this is that the principal's expected profit evaluated at the optimal contracts is a convex piece-wise linear function of v.

Lemma 4. *Suppose the following condition holds.*

$$\frac{W_k^s}{W_n^s} > \frac{p(1_k) - p(1_0)}{p(1_n) - p(1_0)}, \quad \forall k \in \{0, \ldots, n-1\}. \tag{CPs}$$

Then there exists a cutoff

$$v_{0n}^s \equiv \frac{cn}{p(1_n) - p(1_{n-1})}\frac{p(1_n)}{p(1_n) - p(1_0)}$$

such that for all $v > v_{0n}^s$, the principal implements high effort in each task, for $v < v_{0n}$, he implements low effort in each task and for $v = v_{0n}^s$, he is indifferent between implementing high effort in each task and no effort in each task.

This result establishes that if total compensation costs when k agents are induced to work hard relative to that when n agents are induced to work hard is greater than the proportional increase (relative to that when low effort is chosen in each task) in the probability of success when the principal moves from choosing a situation in which he induces high effort in k tasks to one in which he induces high effort in n tasks, there is a cutoff v_{0n}^s such that the principal prefers to induce everyone to work hard whenever the outcome after success v exceeds v_{0n}^s and not to work hard otherwise.

To better understand what this means, it is useful to write condition CPs as follows,

$$\frac{p(1_n) - p(1_0)}{n}\frac{p(1_k)}{\Delta(1_{k-1})} > \frac{p(1_k) - p(1_0)}{k}\frac{p(1_n)}{\Delta(1_{n-1})}.$$

One can easily show that supermodularity is not sufficient for this condition to hold. Yet, log-supermodularity implies this condition holds since $\frac{p(1_k)}{\Delta(1_{k-1})} > \frac{p(1_{k-1})}{\Delta(1_{k-2})}$, $\forall k$. Thus, a strong sort of complementarity is sufficient for having the optimality of e to be reduced to the choice of an effort profile in the set $\{1_0, 1_n\}$.

Lemma 5. *Suppose the following condition holds*

$$\frac{W_{k+1}^s - W_k^s}{W_k^s - W_{k-1}^s} > \frac{p(1_{k+1}) - p(1_k)}{p(1_k) - p(1_{k-1})}. \tag{NPs}$$

Then for each $k \in \{0, \ldots, n-1\}$, there exist cutoffs $v_{k,k-1}^s$ and $v_{k+1,k}^s$ such that for all $v_{k+1,k}^s > v \geq v_{k,k-1}^s$, the principal chooses to implement high effort in k tasks.

This lemma establishes that when condition (NPs) holds, the principal will provide incentives for hard work to exactly k agents when the outcome after success satisfies $v_{k+1,k}^s > v \geq v_{k,k-1}^s$. Thus, as v raises more and more agents are provided with incentives for hard work in their respective tasks.

Babaioff et al. (2012) show that neither SSBM is sufficient for (NPs) nor this for SSBM. Furthermore they show that a technology in which tasks are perfect substitutes is sufficient for (NPs) to hold. This sheds some light on when it is the case that a firm with substitute tasks will choose to induce more and more agents to work hard as v rises.

3.2.2. Multitasking.

Lets define total compensation costs when $U_j(w, e)$ satisfies SSPM and agent j is responsible for each task and works hard in each of them by

$$W^m \equiv p(1_n)B_{sspm} = \frac{p(1_n)}{p(1_n) - p(1_0)}cn.$$

This is defined only for the effort profile $e = 1_n$, since when the success function satisfies SSPM, the principal can implement either $e = 1_n$ or $e = 1_0$. In the latter case total compensation costs are zero. Hence, this is the limited liability rent needed to induce the agent to work hard in each of the n tasks.

Next lets define total compensation costs when $U_j(w, e)$ satisfies SSBM and agent j is responsible for each task and works hard in $k \in \{0, \ldots, n\}$ of them by

$$W_k^m \equiv p(1_k)B_k = \frac{p(1_k)}{\Delta(1_{k-1})}c.$$

This is the limited liability rent needed to induce the agent to work hard in k tasks.

The next lemma is identical to lemma 3. Hence, under the optimal contract, the principal shares the increase in the return after success v with the agent in such a way that both, the principal and agent are better-off.

Lemma 6. *For any technology $p(e)$ each of the following is monotonically non-decreasing with the return v: (i) the expected utility of the principal evaluated at the optimal contracts; (ii) the success probability evaluated at the optimal contracts; and (iii) the expected payment evaluated at the optimal contracts.*

The intuition here is exactly the same as that for the case of specialization; the principal's expected profit evaluated at the optimal contracts is a convex piece-wise linear function of v.

Lets define the return level v_{0n}^{mc} as the lowest return for which the principal is indifferent between implementing 1_n and implementing 1_0; that is, v_{0n}^{mc} is the lowest v such that $V^m(1_n) \geq V^m(1_0)$.

Lemma 7.

Suppose that $U_j(w, e)$ satisfies SSPM. Then there exists a cutoff

$$v_{0n}^{mc} \equiv \frac{cn}{p(1_n) - p(1_0)} \frac{p(1_n)}{p(1_n) - p(1_0)}$$

such that for all $v > v_{0n}^{mc}$, the principal chooses to implement high effort in each task, for $v < v_{0n}^{mc}$, the principal chooses to implement low effort in each task and for $v = v_{0n}^{mc}$, the principal is indifferent between implementing high effort in each task and no effort in each of them.

This shows that when $U_j(w, e)$ satisfies SSPM, it is optimal to implement 1_n when v is large and 1_0 otherwise. In contrast to the result in lemma 4 this does not require condition CP to hold. The reason stands for the fact that when when the probability of success satisfies SSPM and a multitasking assignment is chosen, there is an implementation problem that restricts the set of implementable effort profiles to the ones in which the agent works hard in each task or chooses low effort in each of them. Thus, complementarity in this case is sufficient for the principal to be faced with the problem of choosing between 1_0 and 1_n.

Lets define the return level v_{0n}^{ms} as the lowest return for which the principal is indifferent between implementing 1_n and implementing 1_0; that is, v_{0n}^{ms} is the lowest v such that $V^m(1_n) \geq V^m(1_0)$.

Lemma 8. *Suppose $U_j(w,e)$ satisfies SSBM and the following condition holds*

$$\frac{W_k^m}{W_n^m} > \frac{p(1_k) - p(1_0)}{p(1_n) - p(1_0)}. \tag{CPm}$$

Then there exists a cutoff

$$v_{0n}^{ms} \equiv \frac{c}{p(1_n) - p(1_{n-1})} \frac{p(1_n)}{p(1_n) - p(1_0)}$$

such that for all $v > v_{0n}^{ms}$, the principal chooses to implement high effort in each task, for $v < v_{0n}^{ms}$, the principal chooses to implement low effort in each task and for $v = v_{0n}^{ms}$, the principal is indifferent between implementing high effort in each task and no effort in each task.

When $U_j(w,e)$ satisfies SSBM, this result establishes that if total compensation costs when the agent is induced to work hard in k tasks relative to that when the agent is incentivized to work hard in n tasks is greater than the proportional increase (relative to that when low effort is chosen in each task) in the probability of success when the principal moves from choosing a situation in which he induces high effort in k tasks to one in which he induces high effort in n tasks, there is a cutoff v_{0n}^m such that the principal prefers to induce everyone to work hard whenever the outcome after success v exceeds v_{0n}^m.

Lemma 9. *Suppose $U_j(w,e)$ satisfies SSBM and the following condition holds*

$$\frac{W_{k+1}^m - W_k^m}{W_k^m - W_{k-1}^m} > \frac{p(1_{k+1}) - p(1_k)}{p(1_k) - p(1_{k-1})}. \tag{NPm}$$

Then for each $k \in \{0, \dots, n-1\}$, there exist cutoffs $v_{k,k-1}^m$ and $v_{k+1,k}^m$ such that for all $v_{k+1,k}^m > v \geq v_{k,k-1}^m$, the principal chooses to implement high effort in k tasks.

This lemma establishes that when condition (NPm) holds, the principal will provide incentives for hard work exactly in k tasks when the outcome after success satisfies $v_{k+1,k}^m > v \geq v_{k,k-1}^m$. Thus, as v raises the agent is induced to work hard in more tasks.

In this case condition (NPm) can be written as follows:

$$\frac{p(1_{k+1})\triangle(k-1) - p(1_k)\triangle(k)}{p(1_k)\triangle(k-1) - p(1_{k-1})\triangle(k-2)} > \frac{\triangle(k)^2}{\triangle(k-1)\triangle(k-2)}.$$

Because SSBM implies that $\triangle(k-2) > \triangle(k-1) > \triangle(k)$, the numerator on the LHS is greater than $\triangle(k)^2$ and the denominator is smaller than $\triangle(k)\triangle(k-1)$. Hence, SSBM is sufficient condition for condition (NPm) to hold. This implies that when tasks are substitutes the principal chooses to induce high effort in number of tasks that is strictly increasing with the return v, while when tasks are complements, the principal induces the agent to work hard in every task when the return is high enough and in no task otherwise. Hence, when the probability of success satisfies SSPM, condition (CPm) never holds.

4. Optimal Task Assignment

The principal chooses to assign the responsibility for the n tasks to agent j if and only if

$$V^m(e^m) \geq V^s(e^s).$$

Hence, the following results follows from the lemmas above.

Proposition 1. *Suppose agents are compensated according to an aggregated performance measure and $p(e)$ satisfies SSPM. Then,*

i) *Suppose condition* (CPs) *holds. Then, for all $v \geq v_{0n}^{mc}$, the optimal assignment is multitasking and the principal implements the effort profile 1_n, while for all $v < v_{0n}^{mc}$, the optimal assignment is multitasking and the principal implements the effort profile 1_0.*

ii) *Suppose condition* (NPs) *holds. Then, for all $v \geq v_{0n}^{mc}$, the optimal assignment is multitasking and the principal implements the effort profile 1_n, while for all $v < v_{0n}^{mc}$, the optimal task assignment is specialization and the principal implements the effort profile 1_k, with $k \leq n-1$, if v is such that $v_{k+1,k}^{s} > v \geq v_{k,k-1}^{s}$.*

First I will provide the intuition for the case in which (CPs) holds. The intuition is twofold: (i) in this case and regardless of the task assignment chosen, it is optimal to implement 1_n when v is large and 1_0 otherwise and therefore the implementation problem is irrelevant from the optimality point of view; (ii) providing incentives for $e = 1_n$ is more expensive under specialization. The reason stands for the fact that for any effort profile the sum of the agents' limited liability rents is greater than the limited liability rent for one agent since, under multitasking, the agent internalizes the spillovers that more effort in one task has over the return to effort in other tasks. Hence, whenever $v \geq v_{0n}^{s}$, multitasking has a cost advantage over specialization. When $v_{0n}^{s} > v \geq v_{0n}^{m}$, the cost advantages not only makes the implementation of 1_n cheaper, but it makes this optimal under multitasking and non-optimal under specialization. Finally, when $v < v_{0n}^{s}$, the return is such that implementing 1_0 under either task assignment is optimal.

This results is similar in spirit to that in Laux (2001). He shows that when tasks are independent, there is one performance measure per task, agents are subject to limited liability and the implementation of 1_n is required, multitasking always dominates specialization. The reason stands for the fact under under multitasking the principal can punish failure in one task by not paying a bonus in successful tasks, while under independent contracting this is not possible. This results in a lower limited liability rent when 1_n is implemented.[19] However, the intuition here is rather different since there is an aggregated performance measure and therefore either all tasks fail or succeed simultaneously. This implies that all agents are simultaneously rewarded or punished. In addition, the result here is more general in the sense that it does not depend on the principal wanting to implement the action profile 1_n and on tasks being independent.

Next, lets suppose that condition (NPs) holds. In this case, when a multitasking assignment is chosen, the optimal contract implements 1_n when the return v is sufficiently large and implements 1_0 otherwise. The reason stands for the fact that tasks are complements. In contrast, when there is one task per-agent, there is no implementation problem since there are as many instruments as tasks. The intuition is simple. If each agent carries one task only and agents are compensated according to an aggregated performance measure, each agent can affect his payoff by changing the effort in his own task. The principal then influences the effort choice of the agent by adjusting the pay to this agent. Hence, the optimal contract is such that the principal implements high effort in more tasks as v raises.[20] When v is such that it is optimal to implement 1_n under multitasking, this task assignment dominates specialization for the same reasons given above. In contrast when it is optimal to implement 1_0 under multitasking, specialization dominates multitasking since it allows the implementation of effort profiles where high effort is chosen in $k > 0$ tasks as long as $v_{k+1,k}^{s} > v \geq v_{k,k-1}^{s}$ for $k > 0$. Hence, specialization dominates multitasking because of the implementation problem that arises when an agent is assigned the responsibility in each of the n complementary tasks and the return after success is not sufficiently large to make the effort profile 1_n second-best optimal under multitasking.

Now I consider the case in which tasks are substitutes; i.e., $p(e)$ satisfies SSBM.

[19] Technically, under Laux's assumptions and under the optimal contract $U_j(w, e)$ satisfies SSPM.

[20] Technically, the principal's expected payoff is a convex piece-linear function in which different effort levels are optimal in each different segment.

Proposition 2. *Suppose agents are compensated according to an aggregated performance measure and* $p(e)$ *satisfies SSBM. Then,*

i) *Suppose condition* (CPs) *holds. Then, the principal prefers multitasking to specialization and implements the effort profile* 1_k *if* v *is such that* $v^m_{k+1,k} > v \geq v^m_{k,k-1}$.

ii) *Suppose condition* (NPs) *holds. Then, the principal prefers multitasking to specialization and implements the effort profile* 1_k *if* v *is such that* $v^m_{k+1,k} > v \geq v^m_{k,k-1}$.

This shows that when tasks are substitutes, the principal always prefer multitasking to specialization. The reason is straightforward. When the principal delegates the n tasks to one agent, the implementation of the effort profile 1_k, $k \in \{0, \ldots, n\}$, at the minimum costs requires that the downward local incentive compatibility constraint regarding the effort profile 1_k holds with equality. This incentive compatibility constraint is identical to the one that arises under specialization when the principal wants to induce the agent responsible for task k to choose high effort. Hence the limited liability rent paid under multitasking is $p(1_k)B_k$ and the sum of the limited liability rents paid under specialization is $p(1_k)kB_k$. It follows from this that the principal's cost of implementing 1_k under specialization is k times that under multitasking. In other words, the cost of motivating k agents to work hard is k times higher than motivating one agent to work hard in k tasks.

This results establishes that when there is a multitasking problem in the sense that under multitasking an effort profile other than 1_n cannot be implemented, there are parameterizations such that specialization is preferred to multitasking. Nonetheless, specialization makes the implementation of any effort profile more expensive, which makes the equilibrium effort profile different from the first-best efficient profile implemented when there is no limited liability.

It also worthwhile to comment on the fact that the result is driven by the implementation problem under multitasking. One might conjecture that this is the result of the fact that effort is dichotomic and the performance measure takes two values only. However this conjecture is not correct. Whenever the agent is compensated according to an aggregated performance measure and tasks are complements, there would be an implementation problem. By that I mean that there are effort profiles that are implementable under specialization that cannot be implemented when a multitasking assignment is chosen.[21] The reason is that for any contract that pays more when more is produced, a higher bonus for any given outcome will increase effort in each task since the marginal return to effort in one task increases as the effort in the other task rises. In fact one can show that if tasks enter symmetrically in $p(e)$, only symmetric effort profiles are implementable, while under specialization this is not the case.[22]

5. Robustness

In this section I study the robustness of the implementation problem to richer environments. This section's goal is to show that the implementation problem, which is at the crux of the results in this paper, is not an artifact of the stylized features of the environment studied so far. See, Appendix AppendixB a more detailed analysis of the extensions studied in this section.

5.1. Limited Effort

In this sub-section I study a situation in which there is a maximum number of tasks for which an agent may exert high effort. In particular, I assume that no agent can exert high effort in more than $m < n$ tasks. Because tasks are symmetric, how an agent allocates the m units of effort among the tasks under his responsibility is irrelevant. This constraint has no bite under specialization since each agent is responsible for one task only and when it is second-best

[21] For a formal proof of this for the case of two tasks, continuous effort and multiple outcomes see Balmaceda (2011).

[22] A potential solution to this problem could be the use of stochastic contracts, however this may not come at a low cost.

optimal to implement high effort in $k \leq m$ tasks. So, I will consider parameterizations where the principal wishes to implement high effort in k tasks with $k > m$. To simplify the analysis I will assume that n/m is a natural number and I will denote this by x.

The point I make here is twofold. First, I will claim that it is optimal to allocate the k tasks between the lowest number of agents possible and second, tasks should be split as even as possible among the agents. To see the former lets assume that there is one agent responsible for $x < m$ tasks and another one responsible for $y \leq m$ tasks. Then one can easily show that if tasks are complements, inducing these two agents to work hard in each of the tasks they are responsible for as Nash equilibrium requires to pay them the following total limited liability rent[23]

$$p(1_k)\Big(\frac{cx}{p(1_k) - p(1_{k-x})} + \frac{cy}{p(1_k) - p(1_{k-y})}\Big). \tag{6}$$

Instead if the first agent is in charge of m tasks and the other agent of the remaining tasks $\max\{0, y + x - m\}$, then inducing these two agents to work hard as Nash equilibrium requires to pay them the following total limited liability rent

$$p(1_k)\Big(\frac{c\min\{y + x, m\}}{p(1_k) - p(1_{k-\min\{y+x,m\}})} + \frac{c\max\{0, y + x - m\}}{p(1_k) - p(1_{k-\max\{0,y+x-m\}})}\Big). \tag{7}$$

It is straightforward to check that the limited liability rent in equation (6) exceeds that in equation (7) and therefore whenever there are two agents with different loads and working hard in each task, it is worthwhile to take away tasks from the relatively overloaded agent and to give them to the relatively underloaded agent. Hence, tasks must be split as evenly as possible. This together with the fact that the sum of the limited liability rents is higher than the limited liability rent of one agent in charge of the same tasks implies that it is optimal to have the lower number of agents in charge of the n tasks and these must be split as evenly as possible.

If tasks are substitutes, inducing the two agents responsible for x and y tasks to work hard in each of them as Nash equilibrium requires to pay them the following total limited liability rent[24]

$$p(1_k)\Big(\frac{c}{p(1_k) - p(1_{k-1})} + \frac{c}{p(1_k) - p(1_{k-1})}\Big). \tag{8}$$

Instead if the first agent is in charge of m tasks and the other agent of the remaining tasks $\max\{0, y + x - m\}$, then inducing these two agents to work hard as Nash equilibrium requires to pay them the following total limited liability rent

$$p(1_k)\Big(\frac{c}{p(1_k) - p(1_{k-1})} + \frac{c}{p(1_k) - p(1_{k-1})}\Big). \tag{9}$$

It is easy to see that that the limited liability rent in equation (8) is identical to that in equation (9) and therefore whenever there are two agents with different loads and working hard in each task, the distribution of tasks among them is irrelevant. This together with the fact that the sum of the limited liability rents is higher than the limited liability rent of one agent in charge of the same tasks implies that it is optimal to have the lower number of agents in charge of the n tasks. Hence, the optimal task assignment requires to have the least number of agents possible. This requires that each agent be assigned the greatest number of tasks possible.

Hence, the main trade-off behind the results in this paper is robust to introduction of a total effort constraint that limits an agent's ability to choose high effort in each possible task, since the implementation problem survives to this constraint. Furthermore, a new insight arises which is that it is optimal to allocate the tasks between the smallest number of agents possible and when tasks are complements they must be split as evenly as possible among those agents, while when tasks are substitutes the distribution among them is irrelevant. Hence, when tasks are complements

[23]This is due to the fact that complementarity implies that the global incentive constraint binds.

[24]This is due to the fact that substitutability implies that the only the downward local incentive constraint binds.

and specialization is not optimal, the optimal allocation of tasks will require teams of the smallest feasible size and each member of the team should have the same load in terms of the number of tasks he is responsible for.

5.2. Multiple Performance Measures

In this sub-section I consider a situation where there are multiple aggregated performance measures such as total revenue, stock market value, EBITDA so on and so forth. Lets assume that besides the output there is another performance measures that can take also two outcomes: success with probability $q(e)$ and failure with probability $1 - q(e)$. If $q(e)$ inherits the properties of $p(e)$ in terms of how effort is related across tasks, one can argue that due to the informativeness principle, having multiple performance measures could improve incentives in the sense that provided that an effort profile is implementable, it could be implemented at lower cost. In the case of a multitasking job, the firm will choose to compensate the worker using only the more informative performance measure, while under specialization or team work the solution is not as straightforward since the principal can try to exploit some sort of relative performance evaluation understood of as competition of the team against the team itself. One can show in fact that the optimal contract when the principal wants to implement the effort profile 1_k are as follows: for those workers the principal chooses not to motivate them to work hard, the contract offers to pay zero regardless of the performance measure realizations; and for those whom the principal chooses to incentivize so that they work hard, the contract pays nothing when either performance measure shows a failure and when neither shows a success and a positive bonus only when both realizations are success.

The main lesson to be learned from this is that having multiple performance measures does not help to solve the implementation problem that precludes the optimality of multitasking under certain parameterizations. Consistent with the informativeness principle, having more than one performance measure is beneficial since it allows the principal to implement any implementable effort profile at a lower cost than when there is just one performance measure.

5.3. Contractible Tasks

In this sub-section I assume the principal can costlessly and perfectly observe some, but not all, effort levels. In addition, effort is not only observable, but also verifiable in these tasks. Hence, the principal can contract directly on them. The number of tasks where effort is observable is given by h, $h \in \{1, \ldots, n-1\}$.

In this case the principal can use forcing contracts and punish an agent responsible for a task in which effort is verifiable if he does not exert the required effort level. The magnitude of the punishment is still limited by the limited liability constraint. Hence, when a specialized task assignment is chosen, the principal can implement the desired effort level in those tasks where effort is contractible at no extra cost. This requires to pay each of these agents a fixed wage $\alpha = c$ when $e = 1$ and to pay cero otherwise. When there is multitasking this is no longer the case. Under SSPM the observability of effort in certain tasks allows the principal to push the limited liability rent all the way down to zero since the contractible effort generates informational synergies among tasks. This occurs when the average contribution of effort when high effort is chosen in n tasks is higher than that when high effort is chosen only on the tasks where effort is contractible. This is consistent with the literature on moral hazard that states that any informative signal is worthwhile using in the optimal contract since it allows the principal to lower the cost of implementing a given effort level. Under SSBM one can easily show that the optimal effort is determined by the local incentive constraints. Not only so, but also that the least costly way of inducing high effort in k tasks requires to satisfy the local downward incentive constraint. Hence, the analysis made in section 4 applies directly to this case.

It readily follows from this that contractibility of efforts favor multitasking since the agent in charge of the n tasks internalizes the costs of being punished from not working hard in the tasks were effort is contractible and high effort is requested. In contrast, under specialization the agents responsible for the tasks for which effort is non-contractible play the same simultaneous effort choice game and therefore their best responses are identical to the ones they have when effort cannot be contracted on in any task.

6. Final Remarks

In this paper, I have studied an old question which is what are the benefits of specialization vis-a-vis multitasking. I have done that in a setting where agents are compensated according to an aggregated performance measure, there is moral hazard and agents are subject to limited liability. The main insight of the paper is that multitasking results in a natural source of economies of scope in firms regardless of the technological relationship between tasks. Yet multitasking, together with task complementarity, results in an implementation problem that results in the impossibility to implement an effort profile different from one in which the agent chooses no effort in each task or one in which he chooses high effort in each task. This implementation problem gives rise to the optimality of specialization when the project's return is such that implementing high effort in each task is not second-best optimal. Hence, the technological relationship between tasks not only shape optimal contracts, but also the way firms organize themselves.

One limitation of this paper is that I have not consider other possible task assignments. However, under complementarity this is less important since the implementation problem highlighted here will be present whenever an agent is allocated more than one task. Finally, I hope that this framework and its insights will provide a useful building block for future research drawing out the wider implications of task assignments e.g., for employers deciding whether to use teams in the workplace and designing optimal team incentive schemes, for policy-makers deciding how to tax partnerships and team-based bonuses, and for workers themselves deciding whether or not to join teams.

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AppendixA. Main Proofs

Proof of lemma 6. Lets define the set $A = \{i \in \{0,\dots,n\} | e_i = 1\}$. This is the set of tasks in which the agent chooses high effort. Thus choosing the optimal effort is equivalent to choose the set A. I call the set $A(v)$ as the optimal set of tasks in which the agent is induced to choose high effort when the return after success is v. Let $W(A)$ be the expected total costs for a set A and $e(A)$ the optimal effort profile when set A is chosen. This is given by W_k^m.

The the principal expected utility is given by $V(A,v) \equiv p(e(A))v - W^s(A)$. Lets assume that $v > v'$. Then, because $A(v)$ is optimal at v, $V(A(v),v) > V(A(v'),v)$, while because $v > v'$ and $p(A) > 0$, $V(A(v'),v) > V(A(v'),v')$. It follows from this that $V(A(v),v) > V(A(v'),v')$. Next I show that the success probability is non-decreasing in the value v. Because $A(v)$ is optimal at v,

$$p(e(A(v)))v - W^s(A(v)) \geq p(e(A(v')))v - W^s(A(v')).$$

Because $A(v')$ is optimal at v',

$$p(e(A(v')))v' - W^s(A(v')) \geq p(e(A(v)))v' - W^s(A(v)).$$

Summing these two equations one gets that $\big(p(e(A(v))) - p(e(A(v')))\big)\big(v - v'\big) \geq 0$, which implies that $p(e(A(v))) \geq p(e(A(v'))$ since $v > v'$. Finally, I need to show that the expected total payment is non-decreasing in the value v. Since $A(v')$ is optimal at v' and $p(e(A(v))) \geq p(e(A(v'))$, one gets that

$$p(e(A(v')))v' - W^s(A(v')) \geq p(e(A(v)))v' - W^s(A(v)) \geq p(e(A(v')))v' - W^s(A(v))$$

or equivalently $W^s(A(v)) \geq W^s(A(v'))$, which proves the statement.

□

Proof of lemma 8. Lets define the principal expected payoff when the return after success is v and he optimally induces high effort in k tasks as $V(v,k)$. Let v_{0n} be the value of v that satisfies $V(v,0) = V(v,n)$. It follows from this that $v_{0n} = W_n^m/(p(1_n) - p(1_0))$. Now consider any $k \in \{1,\dots,n-1\}$ and observe that $V(v_{0n},n) > V(v_{0n},k)$ if and only if

$$v_{0n}(p(1_n) - p(1_k)) > W_n^m - W_k^m.$$

Using the definition of v_{0n} and after a few steps of simple algebra, this re-writes as follows

$$\frac{W_k^m}{W_n^m} > \frac{p(1_k) - p(1_0)}{p(1_n) - p(1_0)}.$$

It remains to prove that $V(v,k)$ is maximized by either $k = 0$ or $k = n$ if and only if $V(v_{0n},0) = V(v_{0n},n) > V(v_{0n},k)$ for any $k \in \{1,\dots,n-1\}$.

If part: suppose that $V(v_{0n},n) > V(v_{0n},k)$. Because $p(1_n) > p(1_k)$, $V(v_{0n},n) > V(v_{0n},k)$ implies that for all $v > v_{0n}$ the contract that induces high effort in each task is optimal. Because $p(1_0) < p(1_k)$ for any $k \in \{1,\dots,n-1\}$, $V(v_{0n},0) > V(v_{0n},k)$ implies that for every $v < v_{0n}$, the contract that implements no effort in each task is optimal.

Only if part: Suppose that $V(v,k)$ is maximized by $k = 0$ for all $v < v_{0n}$ and by $k = n$ for all $v > v_{0n}$. Then, inducing high effort in $k \neq \{0,n\}$ tasks is not optimal for any v, it is not optimal for $v = v_{0n}$. Thus, $V(v_{0n},0) = V(v_{0n},n) > V(v_{0n},k)$ for any $k \in \{1,\dots,n-1\}$ as we need to show. □

Proof of lemma 9. Lets define the principal expected payoff when the return after success is v and he optimally induces high effort in k tasks as $V(v,k)$. Let $v_{k,k-1}$ be the value of v that satisfies $V(v,k) = V(v,k-1)$. It follows from this that $v_{k,k-1} = (W_k^m - W_{k-1}^m)/(p(1_k) - p(1_{k-1}))$. Observe now that for every $k \in \{1, \ldots, n-1\}$, $V(v_{k,k-1}, k) > V(v_{k+1,k-1}, k-1)$ if and only if

$$\frac{W_{k+1}^m - W_k^m}{W_k^m - W_{k-1}^m} > \frac{p(1_{k+1}) - p(1_k)}{p(1_k) - p(1_{k-1})}.$$

This follows from substituting the definition of $v_{k,k-1}$ into $V(v_{k,k-1}, k) > V(v_{k+1,k-1}, k-1)$ and rearranging terms. Notice that for any $k \in \{1, \ldots, n-1\}$, $p(1_k) > p(1_{k-1})$. This together with condition NPs implies that $v_{k+1,k} > v_{k,k-1}$. It is needed to show that if $v_{k+1,k} > v_{k,k-1}$ for any $k \in \{1, \ldots, n-1\}$, then there exists a value v_k such that $V(v_k, k) > V(v_k, j)$ for every $j \neq k$. I need to prove the claim that for any $v > v_{k,k-1}$ and $j < k$, inducing k agents to work hard gives the principal a higher expected utility than inducing j agents to work hard; that is, $V(v, k) > V(v, j)$ for any $j < k$ and $v > v_{k,k-1}$. By definition of $v_{1,0}$ and the linearity in v, this clearly holds for $k = 1$. Suppose the claim is true for all $k - 1$, which means that for any $v > v_{k-1,k-2}$ and $j < k - 1$, $V(v, k-1) > V(v, j)$. Because $v_{k,k-1} > v_{k-1,k-2}$, to complete the induction step one needs to show that for any value $v > v_{k,k-1}$ and $j < k$, $V(v, k) > V(v, j)$. By definition of $v_{k,k-1}$ and the linearity in v, it holds that $V(v, k) > V(v, k-1)$. By the induction hypothesis for any $v > v_{k-1,k-2}$ and $j < k-1$, it holds that $V(v, k-1) > V(v, j)$. Thus, for any $v > v_{k,k-1} > v_{k-1,k-2}$ and $j < k-1$, it holds that $V(v, k) > V(v, k-1) > V(v, j)$. A similar induction argument show that at any value $v < v_{k+1,k}$ and $j > k$, it holds that $V(v, k) > V(v, j)$. Combining these two claims one deduces that for any value $v \in (v_{k+1,k}, v_{k,k-1})$ and $j \neq k$, $V(v, k) > V(v, j)$ as needed. □

AppendixB. Details Robustness Section

AppendixB.1. Contractible Tasks

In this sub-section I assume the principal can costlessly and perfectly observe some, but not all, effort levels. In addition, effort is not only observable, but also verifiable in these tasks. Hence, the principal can contract directly on them. The number of tasks where effort is observable is given by h, $h \in \{1, \ldots, n-1\}$.

In this case the principal can use forcing contracts and punish an agent responsible for a task in which effort is verifiable if he does not exert the required effort level. The magnitude of the punishment is still limited by the limited liability constraint. Hence, when a specialized task assignment is chosen, the principal can implement the desired effort level in those tasks where effort is contractible at no extra cost. This requires to pay each of these agents a fixed wage $\alpha = c$ when $e = 1$ and to pay cero otherwise. Hence, the total cost of implementing 1_k under specialization is the following:

$$W_k^s \equiv \begin{cases} ck & \text{if } k \leq h \\ ch + \frac{p(1_k)}{\Delta(1_k)} c(k-h) & \text{if } k > h. \end{cases}$$

The first case considers a situation where the principal wants to implement high effort in k tasks and this number is smaller than or equal to the number of tasks in which effort is contractible. The second case entails an environment in which the number of tasks where effort is contractible is smaller than the number of tasks in which the principal wants to implement high effort. The total cost is the first-best efficient cost for the first h tasks plus the sum of the standard limited liability rents for the $k - h$ tasks. Hence, the fact that some tasks are contractible has no effect on the limited liability rent paid to the agents responsible for tasks with non-contractible efforts.

Under multitasking things are slightly more complicated. First, lets consider the case in which $U_j(w, e)$ satisfies SSPM. If the agent is provided with an incentive intensity such that he chooses to exert effort in $h + 1$ tasks, he will also choose to exert effort in the n tasks as before. Hence, it is easy to show that the total cost of implementing 1_k, with $k \in \{0, \ldots, h\} \cup \{n\}$ under specialization is the following:

$$W^m \equiv \begin{cases} ck & \text{if } k \leq h \\ \frac{p(1_n)}{p(1_n)-p(1_h)}c(n-h) & \text{if } k > h \text{ and } \frac{p(1_n)}{p(1_h)} \leq \frac{n}{h} \\ cn & \text{if } k > h \text{ and } \frac{p(1_n)}{p(1_h)} > \frac{n}{h} \end{cases}$$

The first case considers a situation where the principal wants to implement high effort in k tasks and this number is lower than h; the number of tasks for which effort is contractible. The second case deals with a parametrization in which the principal wants to implement 1_n and the agent prefers to work in h of them than slacking-off in each of them. The last one deals with the opposite case, the incentive intensity that induces the agent to work hard in n tasks instead of h tasks is not large enough to induce him to choose hard work in each task over work in none of them. In the second case, the agent must be given a positive limited liability rent in order to be induced to work hard since either h is small or the performance measure is too noisy. Whereas in the third case, the threat of punishing the agent for not working hard in any of the contractible tasks is strong enough to induce him to work hard in each of them. Hence, the observability of effort in certain tasks allows the principal to push the limited liability rent all the way down to zero since the contractible effort generates informational synergies among tasks. This occurs when the average contribution of effort when high effort is chosen in n tasks is higher than that when high effort is chosen only on the tasks where effort is contractible. This is consistent with the literature on moral hazard that states that any informative signal is worthwhile using in the optimal contract since it allows the principal to lower the cost of implementing a given effort level.

Next, lets consider the case in which tasks are substitutes. As before one can easily show that the optimal effort is determined by the local incentive constraints. Not only so, but also that the least costly way of inducing high effort in k tasks requires to satisfy the local downward incentive constraint. Hence, one can show that the total cost of implementing 1_k is the following:

$$W_k^m \equiv \begin{cases} ck & \text{if } k \leq h \\ \frac{p(1_k)}{p(1_k)-p(1_{k-1})}c & \text{if } k > h \text{ and } \frac{p(1_k)}{p(1_{k-1})} \leq \frac{k}{k-1} \\ ck & \text{if } k > h \text{ and } \frac{p(1_k)}{p(1_{k-1})} > \frac{k}{k-1} \end{cases}$$

The rationale for this payment scheme is the same as before with the difference that here the principal can implement any effort profile he wishes.

The analysis made in section 4 applies directly to this case. It is enough to repeat the analysis almost verbatim considering the total compensation costs as derived here. For the sake of brevity, I would not do so. It is easy to see that contractibility of efforts favor multitasking since the agent in charge of the n tasks internalizes the costs of being punished from not working hard in the tasks were effort is contractible and high effort is requested. In contrast, under specialization the agents responsible for the tasks for which effort is non-contractible play the same simultaneous effort choice game and therefore their best responses are identical to the ones they have when effort cannot be contracted on in any task.

AppendixB.2. Multiple Performance Measures

Lets assume that besides the output there is another performance measures that can take also two outcomes: success with probability $q(e)$ and failure with probability $1 - q(e)$.

Lets define the contract $w \equiv (\alpha, \beta_{ss}, \beta_{sf}, \beta_{fs}, \beta_{ff})$, where β_{ss} is the bonus payment when success is observed in both performance measures, β_{sf} is the bonus payment when success in the outcome and failure in the performance measure are realized, β_{fs} is the bonus payment when success in the performance measure and failure in the outcome occur and β_{ff} is the bonus payment when failure in both performance measures is observed.

When an agent faces contract $w \equiv (\alpha, \beta_{ss}, \beta_{sf}, \beta_{fs}, \beta_{ff})$ and chooses the effort profile e, his utility is given by:

$$U(w,e) \equiv \alpha + p(e)(q(e)\beta_{ss} + (1-q(e))\beta_{sf}) + (1-p(e))(q(e)\beta_{fs} + (1-q(e))\beta_{ff}) - c\sum_{i=1}^{n} e_i$$

and the principal's expected payoff is as follows:

$$V(w,e) \equiv p(e)v - \sum_{j=1}^{J} \big(\alpha_j + p(e)(q(e)\beta_{ssj} + (1-q(e))\beta_{sfj}) + (1-p(e))(q(e)\beta_{fsj} + (1-q(e))\beta_{ffj})\big).$$

It is easy check the following

$$\begin{aligned} &p(e)(q(e)\beta_{ss} + (1-q(e))\beta_{sf}) + (1-p(e))(q(e)\beta_{fs} + (1-q(e))\beta_{ff}) \\ =&p(e)q(e)(\beta_{ss} + \beta_{ff} - \beta_{sf} - \beta_{fs}) + p(e)(\beta_{sf} - \beta_{ff}) + q(e)(\beta_{fs} - \beta_{ff}) + \beta_{ff} \end{aligned}$$

It follows from this and the limited liability constraint that an optimal contract entails $\beta_{ff} = 0$, since lowering this decreases the cost of implementing the effort profile e and the incentive compatibility is made to hold by choosing the other payments accordingly. To save on notation lets define $x \equiv \beta_{ss} + \beta_{ff} - \beta_{sf} - \beta_{fs}$, $y \equiv \beta_{sf} - \beta_{ff}$ and $z \equiv \beta_{fs} - \beta_{ff}$.

First, lets consider a multitasking task assignment. It is then easy to check that the existence of two performance measures cannot solve the implementation problem that arises when the technology satisfies SSPM. To see this lets suppose that $p(e)$ and $q(e)$ satisfy SSPM and the principal wants to implement $1_k \notin \{1_0, 1_n\}$. Then, the following must hold

$$(p(1_k)q(1_k) - p(1_{k'})q(1_{k'}))x + (p(1_k) - p(1_{k'}))y + (q(1_k) - q(1_{k'}))z \geq c(k-k'),\ \forall k' \neq k,$$

If $k' = k-1$, the following must hold

$$(p(1_k)q(1_k) - p(1_{k-1})q(1_{k-1}))x + (p(1_k) - p(1_{k-1}))y + (q(1_k) - q(1_{k-1}))z \geq c,$$

while if $k' = k+1$, the following must hold

$$(p(1_k)q(1_k) - p(1_{k+1})q(1_{k+1}))x + (p(1_k) - p(1_{k+1}))y + (q(1_k) - q(1_{k+1}))z \geq -c.$$

Hence, in order to implement $1_k \notin \{1_0, 1_n\}$, a necessary condition is the following

$$\begin{aligned} &(p(1_k)q(1_k) - p(1_{k-1})q(1_{k-1}))x + (p(1_k) - p(1_{k-1}))y + (q(1_k) - q(1_{k-1}))z \geq c \geq \\ &(p(1_{k+1})q(1_{k+1}) - p(1_k)q(1_k))x + (p(1_{k+1}) - p(1_k))y + (q(1_{k+1}) - q(1_k))z. \end{aligned}$$

After some simple algebra this equation re-writes as follows

$$\begin{aligned} &(2p(1_k)q(1_k) - p(1_{k+1})q(1_{k+1}) - p(1_{k-1})q(1_{k-1}))x + \\ &(2p(1_k) - p(1_{k-1}) - p(1_{k+1}))y + (2q(1_k) - q(1_{k-1}) - q(1_{k+1}))z \geq 0. \end{aligned}$$

Because by definition SSPM implies that the second and third term in parenthesis are negative and one can show that

SSPM implies that the first term is negative,[25] this inequality does not hold. If the extra performance measure were to behave differently from the outcome in terms of the relationship between tasks, the implementation problem could be somehow solved. Yet, I will be agnostic with respect to this and I will stick to the natural assumption that $q(e)$ inherits the properties of $p(e)$.

Due to the informativeness principle, having multiple performance measure could improve incentives in the sense that provided that an effort profile is implementable, it could be implemented at lower cost. In the case of a multitasking job, the firm will choose to compensate the worker using only the more informative performance measure, while under specialization or team work the solution is not as straightforward since the principal can try to exploit some sort of relative performance evaluation understood of as competition of the team against the team itself. One can show in fact the optimal contract when the principal wants to implement the effort profile 1_k are as follows: for those workers whom the principal chooses not to motivate them to work hard, the contract offers to pay zero regardless of the performance measure realizations; and for those the principal chooses to incentivize so that they work hard, the contract pays nothing when either performance measure shows a failure and when neither shows a success and a positive bonus only when both realizations are success. The lowest bonus that implements 1_k is equal to

$$\frac{c}{p(1_k)q(1_k) - p(1_{k-1})p(1_{k-1})}.$$

The main lesson to be learned from this is that having multiple performance measures does not help to solve the implementation problem that precludes the optimality of multitasking under certain parameterizations. Consistent with the informativeness principle, having more than one performance measure is beneficial since it allows the principal to implement any implementable effort profile at a lower cost than when there is just one performance measure.

[25] Note that

$$\begin{aligned}&2p(1_k)q(1_k) - p(1_{k+1})q(1_{k+1}) - p(1_{k-1})q(1_{k-1})\\ =&p(1_k)q(1_k) - p(1_{k+1})q(1_{k+1}) + p(1_k)(q(1_k) - q(1_{k-1})) + q(1_{k-1})(p(1_k) - p(1_{k-1}))\end{aligned}$$

SSPM implies that $q(1_{k+1}) - q(1_k) > q(1_k) - q(1_{k-1})$ and $p(1_{k+1}) - p(1_k) > p(1_k) - p(1_{k-1})$. Hence,

$$\begin{aligned}&p(1_k)q(1_k) - p(1_{k+1})q(1_{k+1}) + p(1_k)(q(1_k) - q(1_{k-1})) + q(1_{k-1})(p(1_k) - p(1_{k-1}))\\ <&p(1_k)q(1_k) - p(1_{k+1})q(1_{k+1}) + p(1_k)(q(1_{k+1}) - q(1_k)) + q(1_{k-1})(p(1_{k+1}) - p(1_k))\\ =&(p(1_{k+1}) - p(1_k))(q(1_{k-1}) - q(1_{k+1}))\\ <&0.\end{aligned}$$